Chain rule with power rule

The chain rule is often one of the hardest concepts for calculus students to understand. It's also one of the most important, and it's used all the time, so make sure you don't leave this section without a solid understanding.

Chain rule lets us calculate derivatives of equations made up of nested functions, where one function is the "outside" function and one function is the "inside function. If we have an equation like

$$y = g\left[f(x)\right]$$

then $g\left[f(x)\right]$ is the outside function and $f(x)$ is the inside function. The derivative looks like

$$y' = \left\{g'\left[f(x)\right]\right\}\left[f'(x)\right]$$

Notice here that we took the derivative first of the outside function, $g\left[f(x)\right]$, leaving the inside function, $f(x)$, completely untouched, and then we multiplied our result by the derivative of the inside function.

So applying the chain rule requires just two simple steps.

1. Take the derivative of the "outside" function, leaving the "inside" function untouched.

2. Multiply your result by the derivative of the "inside" function.

Sometimes it's helpful to use substitution to make it easier to think about $g\left[f(x)\right]$. We just replace the inside function with u, and we get

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$$y = g[u]$$

Then the derivative would be

$$y' = g'[u](u')$$

If you're going to use substitution, make sure you back-substitute at the end of the problem to get your final answer.

Example

Use chain rule to find the derivative.

$$y = \left(4x^8 - 6\right)^6$$

Our outside function is $\left(4x^8 - 6\right)^6$, and our inside function is $4x^8 - 6$. Using the substitution method, $u = 4x^8 - 6$ and $u' = 32x^7$.

We'll substitute u into the original equation and get

$$y = (u)^6$$

We'll start to calculate the derivative, and using power rule with chain rule, we find that

$$y' = 6(u)^5(u')$$

Finally, we back-substitute for u and u'.

$$y' = 6\left(4x^8 - 6\right)^5\left(32x^7\right)$$

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$$y' = 192x^7\left(4x^8 - 6\right)^5$$

We just worked an example of chain rule used in conjunction with power rule. We'll also need to know how to use it in combination with product rule, with quotient rule, and with trigonometric functions, which we'll tackle in the next few lessons.

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